Tutorial

Mohr on Receiver Noise

Characterization, Insights & Surprises

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Presented to the

Microwave Theory & Techniques Society

of the

IEEE Long Island Section

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Purpose

The purpose of this presentation is to describe the background and application of the concepts of Noise Figure and Noise Temperature for characterizing the fundamental limitations on the absolute sensitivity of receivers*.

** "Receivers" as used here, is general in sense and the concepts are equally applicable to the individual components in the receiver cascade, both active and passive, as well as to the entire receiver. These would then include, for example, amplifier stages, mixers, filters, attenuators, circuit elements, and other incidental elements.*

Approach

- A step-by-step approach is taken to establish the basis, and concepts for the absolute characterization of the sensitivity of receivers, and to provide the foundation for a solid understanding and working knowledge of the subject.
- Signal quality in a system, as characterized by the signal-to-noise power ratio (S/N), is introduced, but shown to not be a unique characterization of the receiver alone.
- The origin of the major components of receiver noise and their characteristics are then summarized
- Noise Figure, F, is introduced which uniquely characterizes the degradation of S/N in a receiver.
- The precise definitions of Noise Figure and of its component parts are presented and illustrated with models and examples to provide valuable insight into the concepts and applications.
- The formulation for the Noise Figure of a cascade of devices is derived and illustrated by examples.

Approach (Cont'd)

- The concept of Noise Temperature, T_e, is introduced and is shown to be directly derivable from Noise Figure.
- T_e is shown to be a more concise characterization of the receiver alone, by completely eliminating source noise from the equation.
- Its application is illustrated by examples.
- The basic methods of measurement of Noise Figure and Noise Temperature are described and compared.
- Finally, a Summary reviews the material presented, and recommendations are provided for further study.
- References are provided
- An annotated bibliography follows that is intended to serve as a basis for further study

Introduction

Signal/Noise (S/N) Characterization

- Introduction of vacuum tube amplifier
  - Discovered there are limits on achievable signal sensitivity
- Achievable signal sensitivity in a Communication, Radar, or EW receiving system is always limited ambient noise along with the output signal
- Signal quality is characterized by the final output Signal/Noise (S/N) ratio
  - Depending on application, S/N of at least 10 or 20 dB, or more, may be required
  - S/N at the input of a receiver is the best it will be
  - Each component in the receiver cascade, while performing its intended function, degrades the output S/N

Introduction (Cont'd)

Factors Affecting System S/N

- In a communication system, S/N is a function of:
  - Transmitter output power
  - Gain of Transmit and Receive antennas
  - Path loss
  - Receiver noise - the topic of this presentation
- To characterize the receiver alone, Friis(1) introduced Noise Figure which characterized the degradation in S/N by the receiver.
  - Noise Figure of a receiver is the ratio of the S/N at its input to the S/N at its output

Receiver Noise Summary (Cont'd)

Thermal Noise[2,3]

- Thermal noise (Johnson Noise) exists in all resistors and results from the thermal agitation of free electrons therein
  - The noise is white noise (flat with frequency)
  - The power level of the noise is directly proportional to the absolute temperature of the resistor
  - The level is precisely e_n^2=4kTRB (V^2), or 4kTR (V^2/Hz)
    - Where,
      - k is Boltzman's constant =1.38x10^{-23} Joules/ºK
      - T is the absolute temperature of the resistor in ºK
      - R is the value of the resistance in Ohms
      - B is the effective noise bandwidth
  - The available noise power is e_n^2/4R = kTB
  - At T=T_O=290ºK (the standard for the definition of Noise Figure), kT_OB= 4.00x10^{-21} W/Hz (= -204.0 dBW/Hz= -174.0 dBm/Hz = -114.0 dBm/MHz)

Thermal noise in the resistance of the signal source is the fundamental limit on achievable signal sensitivity

Receiver Noise Summary (Cont'd)

Shot Noise[4,7]

- Shot Noise was studied by Schottky, who likened it to shot hitting a target
  - Results from the fluctuations in electrical currents, due to the random passage of discrete electrical charges through the potential barriers in vacuum tubes and P-N junctions
  - Its noise characteristic is white
  - The power level of the noise is proportional to the level of the current through the barrier
  - In vacuum tube diodes, in temperature-limited operation, shot noise is precisely, $i_n^2(f) = 2eI_O (A^2/Hz)$, where I_O is the diode current and e is the electronic charge = 1.6×10^{-19} Coulombs
  - Vacuum tube diodes, in temperature-limited operation, were the first broadband noise sources for measurement of receiver noise figure.

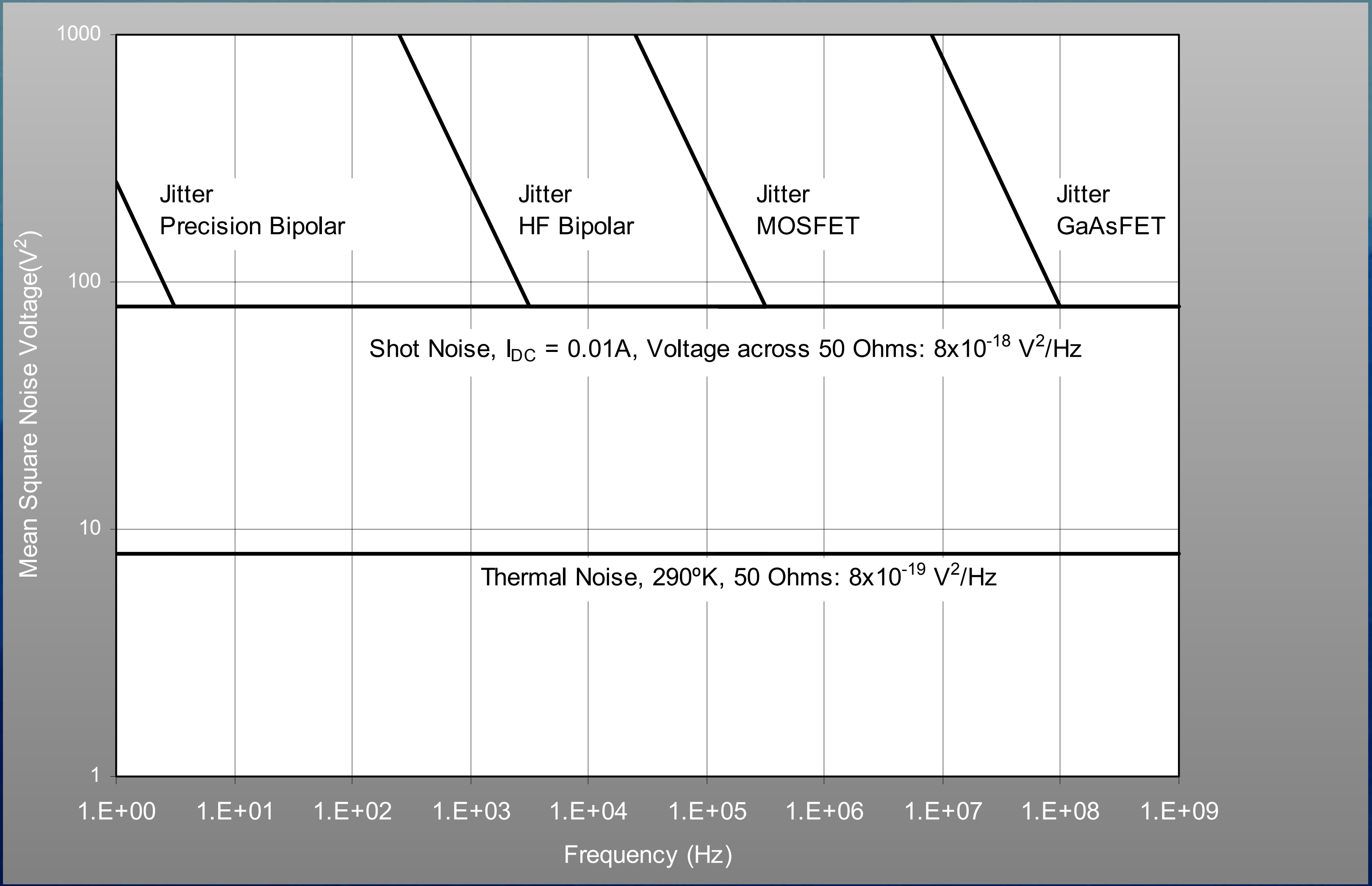
Receiver Noise Summary (Cont'd)

Flicker (1/f) Noise[4,7]

- Flicker Noise appears in vacuum tubes and semiconductor devices at very low frequencies
  - Its origin is believed to be attributable to contaminants and defects in the crystal structure in semiconductors, and in the oxide coating on the cathode of vacuum tube devices
  - Commonly referred to as 1/f noise because of its low-frequency variation
  - Its spectrum rises above the shot noise level below a corner frequency, f_L, which is dependent on the type of device and varies from a few Hz for some bipolar devices to 100 MHz for GaAs FETs

Receiver Noise Summary (Cont'd)

Comparison of Levels of Major Components of Receiver Noise



Noise Figure, F

Definition[*(1)]

$$F = \frac{S_i / N_i}{S_O / N_O} = \frac{N_O}{GN_i} = \frac{N_O}{GkT_O B} = \frac{GkT_O B + N_R}{GkT_O B}$$

- F is the Noise Figure of the receiver
- S_i is the available signal power at the input
- S_O is the available signal power at the output
- $N_i = kT_OB$ is the available noise power at the input
- T_O is the absolute temperature of the source resistance, 290 ºK
- N_O is the available noise power at the output, and includes amplified input noise
- N_R is the noise added by the receiver
- G is the available gain of the receiver
- B is the effective noise bandwidth of the receiver

**The unique and very precise definition of Noise Figure and its component parts makes provision for a degree of mismatch between component parts of a receiver chain which is often necessary for minimum Noise Figure*

Noise Figure

Example, Relating Noise Figure to Sensitivity

What level of input signal, S_i, is required for an output S_O/N_O = 10 dB in a receiver with NF= 6 dB, and B=0.1 MHz?

- From definition of F:

$$F = \frac{N_O}{GkT_O B}$$

and so,

$$N_O = FGkT_O B$$

- Input sensitivity is evaluated by referring the output noise, N_O, to the receiver's input, i.e.

$$N_{Oi} = \frac{N_O}{G} = FkT_O B$$

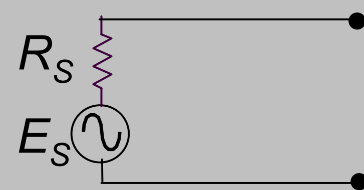
- N_{Oi}(dBm) = NF(dB)+KT_O(dBm/MHz)+10 Log B(MHz) = 6 -114-10 = -118 dBm

- For a desired S_O/N_O of 10 dB, S_i must be at least:
  S_i =-118 dBm+10 dB = -108 dBm

Definition of Factors in Noise Figure

Available Input Signal Power (S_i)

- S_i is the signal power that would be extracted from a signal source by a load conjugately* matched to the output of the source i.e.:



$$S_i = \frac{E_S^2}{4R_S}$$

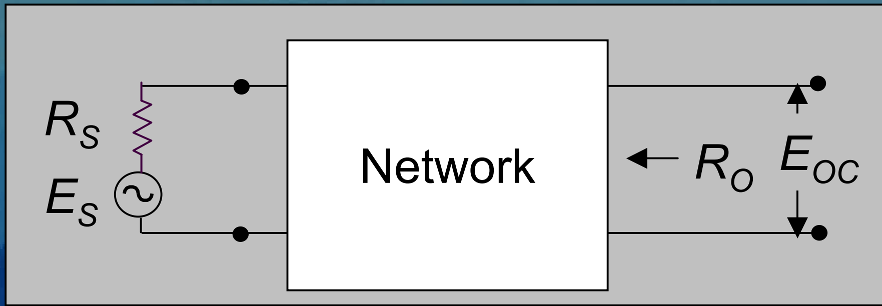
- S_i is dependent only on the characteristics of the source, specifically it is independent of the impedance of the actual load, R_L.
- For a load, $R_L \neq R_S$, the delivered power is less than the available power, but the available power is still S_i.

* *For simplicity and without loss of illustrative value, ideal transformers and reactive elements are not included in models here since they are loss-free and do not directly contribute to receiver Noise Figure*

Definition of Factors in Noise Figure (Cont'd)

Available Output Signal Power, S_O

- S_O is the power that would be extracted by a load conjugately matched to the output of the network, i.e.:



$$S_O = \frac{E_{OC}^2}{4R_O}$$

- S_O is dependent only on the characteristics of the network and its signal source, and the impedance match at its input
- S_O is independent of the actual load, R_L, on the network
  - For a load, $R_L = R_O$, the delivered power will equal the available output power, S_O
  - for $R_L \neq R_O$ the delivered power will be less than the available output power, but the available output power is still S_O
  - Available output is maximum achievable when input is matched to R_S

Definition of Factors in Noise Figure (Cont'd)

Available Gain, G*

$$G = {S_O}/{S_i}$$

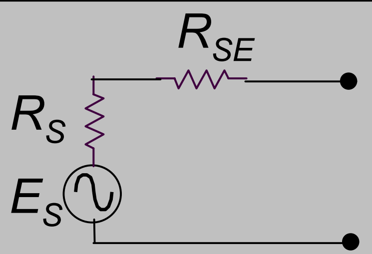
- Definition is applicable to both active and passive devices
- G is independent of impedance match at output
- G is dependent on impedance match at the input
- In general, G is:
  - Less than, or equal to, the maximum available gain
  - Equal to the maximum available gain when source is matched to the input
  - Greater than, or equal to, the insertion gain
  - May be less than, greater than, or equal to, one (unity)

**When used herein, G will always refer to the available gain*

Definition of Factors in Noise Figure (Cont'd)

Available Gains Of Elementary Networks

Gain of Series Resistor

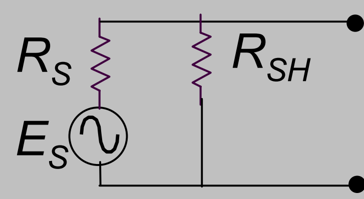


$$G_{SE} = \frac{S_O}{S_i} =$$

$$\frac{E_S^2 \big/ 4(R_S + R_{SE})}{E_S^{\,2} \big/ 4R_S} = \frac{R_S}{R_S + R_{SE}}$$

G_{SE} *is greatest when* $R_S >> R_{SE}$

Gain of Shunt Resistor



$$G_{SH} = \frac{S_O}{S_i} =$$

$$\frac{\left(E_S \frac{R_{SH}}{R_S + R_{SH}}\right)^2 \Big/ 4\frac{R_S R_{SH}}{R_S + R_{SH}}}{E_S^2 / 4R_S} = \frac{R_{SH}}{R_S + R_{SH}}$$

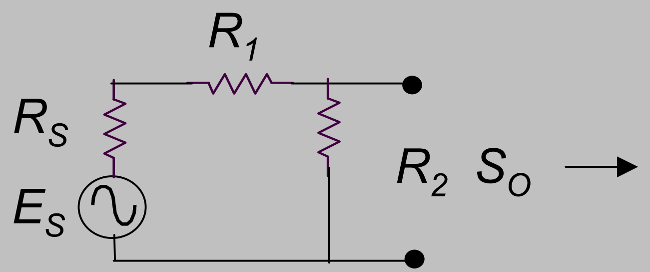
G_{SH} *is greatest when* $R_S << R_{SH}$

The gains of the resistor networks are less than one (1), and are often expressed instead as a power loss ratio, L =1/G, which is then >1; only G will be used here.

Definition of Factors in Noise Figure (Cont'd)

Available Gains Of Elementary Networks (Cont'd)

Resistor L-Section Network



$$S_O = \frac{\left[\dfrac{E_S R_2}{R_S + R_1 + R_2}\right]^2}{4\dfrac{R_2(R_1 + R_S)}{\mathrm{R_S} + R_1 + R_2}},\ S_i = \frac{E_S^2}{4R_S}$$

$$G_L = \frac{S_O}{S_i} = \frac{R_S R_2}{(R_1 + R_S)(R_1 + R_2 + R_S)} = \left(\frac{R_S}{R_1 + R_S}\right)\left(\frac{R_2}{R_1 + R_2 + R_S}\right) = G_{SE} G_{SH}$$

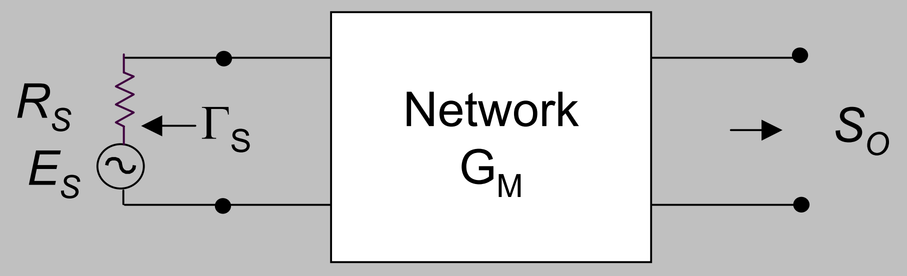
Gain is a maximum with $R_S = \sqrt{R_1(R_1 + R_2)}$

Available gain of L-Section is seen to be the product of the gains of the series section (R_1 and R_S) and the shunt section (R_2 in parallel with series connection of R_S and R_1)

Definition of Factors in Noise Figure (Cont'd)

Available Gains Of Elementary Networks (Cont'd)

2-Port Network



$$G = G_M \frac{1-|\Gamma_S|^2}{1-G_M|\Gamma_S|^2}$$

Where:

- Γ_S is the reflection coefficient of R_S relative to the input characteristic impedance of the network
- G_M is the maximum available gain of the network, i.e. the available gain with a matched input, $\Gamma_S = 0$

Definition of Factors in Noise Figure (Cont'd)

Effective Noise Bandwidth, B

- Noise bandwidth, B, is defined as the equivalent rectangular pass band that passes the same amount of noise power as is passed in the usable receiver band, and that has the same peak in-band gain as the actual device has. It is the same as the integral of the gain of the device over the usable frequency band , i.e.:

$$B = \int_0^\infty \frac{G(f)}{G_O}\, df$$

Where:

- B is the effective noise bandwidth
- G(f) is the gain as a function of frequency over the usable frequency band
- G_O is the peak value of in-band gain

Typically, B is approximately equal to the 3 dB bandwidth.

For greatest sensitivity, B should be no greater than required for the information bandwidth.

Definition of Factors in Noise Figure (Cont'd)

Available Input Noise

- Input noise, N_i, is defined as the thermal noise (Johnson Noise[(2)]) generated in the resistance of the signal source
- The input mean-square noise voltage is expressed concisely as[(3)]:

$$e_{n\,i}^2 = 4kT_O RB$$

Where

- e_{ni}^2 is the mean-square noise voltage of the Thevinin voltage source, in V^2/Hz
- K is Boltzman's constant = 1.38×10^{-23} Joules/ºK
- T is the absolute temperature of the resistor in ºK
- In noise figure analysis, standard temperature is T_O=290ºK
- R is the resistance, in Ohms
- B is the effective noise bandwidth

- The available noise power, N_i (W), from the resistor is:

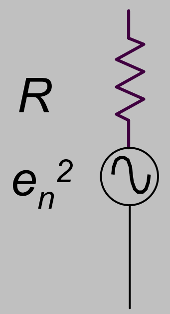
$$N_i = \frac{4kT_O RB}{4R} = kT_O B$$

- With T_O at 290ºK, $N_i = 4.00 \times 10^{-21}$ W/Hz = -204.0 dBW/Hz = -174.0dBm/Hz = -114.0 dBm/MHz

Definition of Factors in Noise Figure (Cont'd)

Thermal Noise Models (Cont'd)

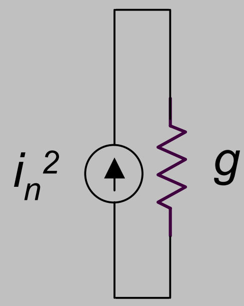
Thevinin Voltage Model



$$e_n^2 = 4kTRB$$

$$N = \frac{4kTRB}{4R} = kTB$$

Norton Current Model



$$i_n^2 = 4KTgB$$

$$N = \frac{4KTgB}{4g} = kTB$$

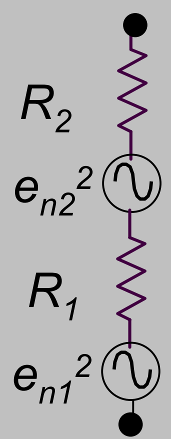
- Where:
  - e_n^2 is the mean-square noise voltage of the Thevinin voltage source, in V^2/Hz
  - i_n^2 is the mean-square noise current of the Norton current source, in A^2/Hz
  - K is Boltzman's constant = 1.38×10^{-23} Joules/ºK
  - T is the absolute temperature of the resistor in ºK
    - In noise figure analysis, standard temperature is T_O=290ºK
  - R is the value of the resistance, in Ohms
  - g is the value of the conductance, in mhos
  - B is the effective noise bandwidth, in Hz
  - N is the available noise power, in W

Definition of Factors in Noise Figure (Cont'd)

Thermal Noise Models (Cont'd)

Will network of resistors provide more noise power than kTB?

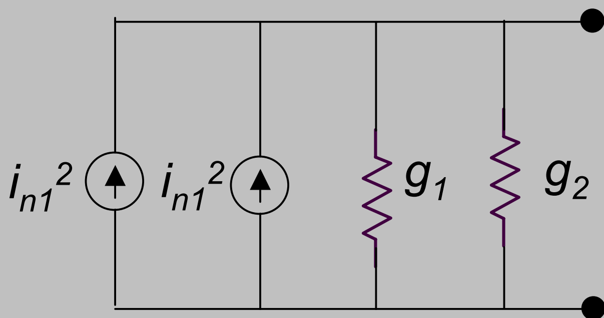
Series Voltage Model



$$e_{n12}^2 = e_{n1}^2 + e_{n2}^2 = 4kTB(R_1 + R_2)$$

$$N_{12} = \frac{4kTB(R_1 + R_2)}{4(R_1 + R_2)} = kTB$$

Shunt Current Model



$$i_{n12}^2 = i_{n1}^2 + i_{n2}^2 = 4kTB(g_1 + g_2)$$

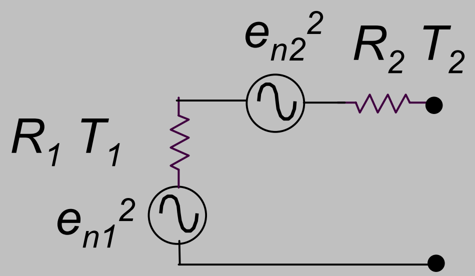
$$N_{12} = \frac{4kTB(g_1 + g_2)}{4(g_1 + g_2)} = kTB$$

Available noise power from network of resistors at T is still just kTB

Definition of Factors in Noise Figure (Cont'd)

Thermal Noise Models (Cont'd)

Series resistors at different temperatures



$$e_{n1-2}^{2} = e_{n1}^{2} + e_{n2}^{2} = 4kB(R_1T_1 + R_2T_2) =$$

$$4kB\left[(R_1 + R_2)T_2 + R_1(T_1 - T_2)\right]$$

$$N_{1-2} = \frac{e_{n1-2}^{2}}{4(R_1 + R_2)} = kT_2B + k(T_1 - T_2)B\frac{R_1}{R_1 + R_2} =$$

$$kT_2B + G_{SE}k(T_1 - T_2)B$$

- When $T_1 = T_2$, the second term in N_{1-2} disappears and $N_{1-2} = kT_2B$
- When $T_1 \neq T_2$, the term $k(T_1-T_2)B$ may be considered the excess available noise power of the source resistor, R_1.
- For noise calculations, the excess available noise power may be treated as though it was signal
- The excess noise power is then attenuated by the term , $R_1/(R_1+R_2)$, which is recognized as the gain of the series resistor, and the attenuated result adds to the kT_2B, power at the output*

**When* $T_1 < T_2$*, then* $N_{1-2} < kT_2B$

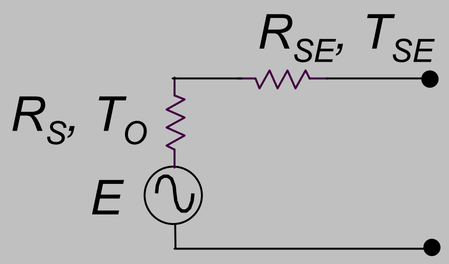
Noise Figure Of Elementary Networks

Optimum Source Impedance for Minimum Noise Figure

- Each component in a receiver cascade can be characterized by an available Noise Figure (F) and available gain (G)
- The available noise figure of each component is dependent only on its source impedance within the receiver chain
- Every component having a noise figure has an optimum noise figure which is achieved when it is supplied from its optimum source impedance
- The optimum source impedance for components is not always the same as required for maximum gain, and so there will be an "optimum input mismatch"
  - In such cases, although the operational available output signal is reduced, the available output noise is reduced proportionally more

Noise Figure Of Elementary Networks (Cont'd)

F of Series Resistor



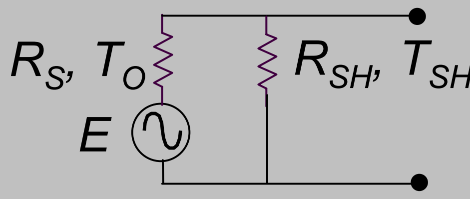
$$F = \frac{N_O}{GkT_O B} = \frac{kT_{SE}B + Gk(T_O - T_{SE})B}{GkT_O B}$$

$$= 1 + \frac{T_{SE}}{T_O}\left[\frac{1}{G} - 1\right]$$

$$\text{Where, } G = \frac{R_S}{R_S + R_{SE}}$$

Noise figure is minimum when $R_S >> R_{SE}$

F of Shunt Resistor



$$F = \frac{N_O}{GkT_O B} = \frac{kT_{SH}B + Gk(T_O - T_{SH})B}{GkT_O B}$$

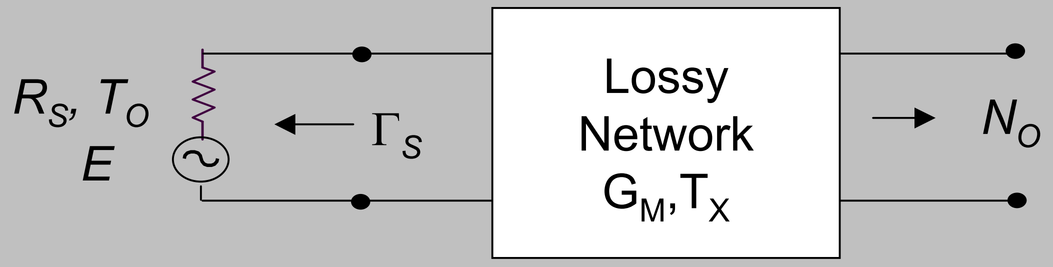
$$= 1 + \frac{T_{SH}}{T_O}\left[\frac{1}{G} - 1\right]$$

$$\text{Where, } G = \frac{R_{SH}}{R_S + R_{SH}}$$

Noise figure is minimum when $R_S << R_{SH}$

Noise Figure Of Elementary Networks (Cont'd)

Attenuators, Lossy Line Sections



$$F = \frac{N_O}{GkT_OB} = \frac{kT_XB + Gk(T_O - T_X)B}{GkT_OB} = 1 + \frac{T_X}{T_O}\left[\frac{1}{G} - 1\right]$$

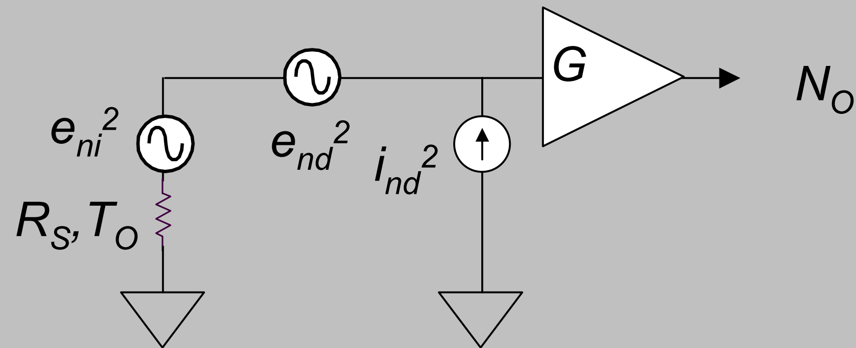
Where G is: $$G = G_M \frac{1 - |\Gamma_S|^2}{1 - G_M|\Gamma_S|^2}$$

By inspection, F is least when G is greatest (R_S is matched to the characteristic input impedance of the network Γ_S =0)

Noise Figure Of Elementary Networks (Cont'd)

Active Devices

Simplified noise model for active devices references all noise sources to input



Output noise power, N_O, referenced to input is:

$$N_{Oi} = \frac{e_{ni}^2 + e_{nd}^2 + i_{nd}^2 R_S^2}{4R_S} = \frac{4kT_o R_S + e_{nd}^2 + i_{nd}^2 R_S^2}{4R_S}$$

$$N_i = kT_O$$

$$F = \frac{N_{Oi}}{N_i} = 1 + \frac{e_{nd}^2 + i_{nd}^2 R_S^2}{4kT_O R_S}$$

$$F \text{ is minimum when } R_S = e_{nd}/i_{nd} \text{ and is, } F = 1 + \frac{e_{nd}^2}{2kT_O R_S}$$

Noise Figure Of Elementary Networks (Cont'd)

Example, Active Device

Find optimum noise figure and required source impedance for amplifier with input voltage and current noise sources for the amplifier specified as:

$e_n^2 = 1.6x10^{-18}$ V^2/Hz and,

$i_n^2 = 4x10^{-23}$ A^2/Hz

F is minimum when $R_S = e_{nd} / i_{nd}$ and is, $F = 1 + \dfrac{e_{nd}^2}{2kT_O R_S}$

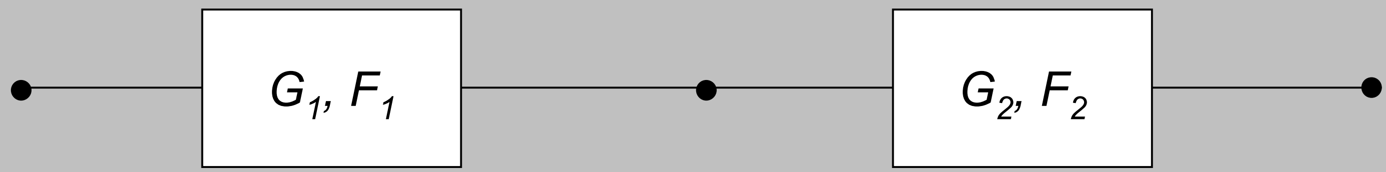
Therefore:

$$R_S = \sqrt{\frac{1.6x10^{-18}}{4x10^{-23}}} = 200\ Ohms$$

$$F = 1 + \frac{1.6\text{x}10^{-18}}{2(1.38\text{x}10^{-23})(290)(200)} = 2;\ \ NF = 3dB$$

Cascade Formula for Noise Figure

Noise Figure, F for cascade of 2 or more devices



S_i	$G_1 S_i$	$G_1 G_2 S_i$
$KT_O B$	$N_{O1} = F_1 G_1 K T_O B$	$N_{O2} = G_2 N_{01} + N_{R2} = G_1 G_2 F_1 k T_O B + (F_2 - 1) G_2 k T_O B$

$$F_{12} = \left(\frac{1}{G_1 G_2}\right)\left(\frac{G_1 G_2 F_1 k T_O B + (F_2 - 1) G_2 k T_O B}{k T_O B}\right) = F_1 + \frac{F_2 - 1}{G_1}$$

In general, for cascade of n devices:

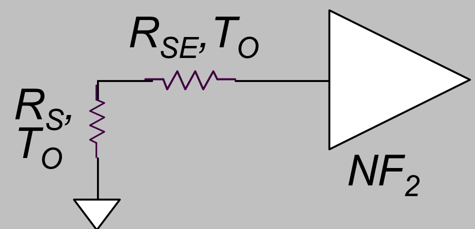
$$F_{1-n} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots + \frac{F_n - 1}{G_1 G_2 \ldots G_{n-1}}$$

Cascade Formula for Noise Figure (Cont'd)

Optimization Example

In the example on Slide 28, it was determined that the amplifier has an optimum noise figure, NF, of 3 dB when operated from a source impedance of 200 Ohms. When operating from a source of 50 Ohms, what is the best way to optimize for best noise figure? Using the cascade approach, the analyses below illustrate two optimization approaches. Which is best?

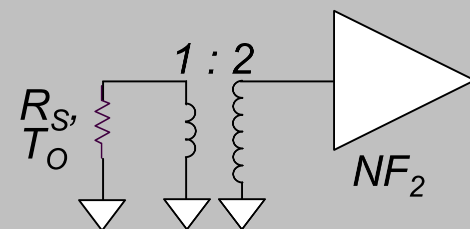
Optimize with series 150 Ohm resistor



$$F_{1-2} = F_1 + \frac{F_2 - 1}{G_1} = \frac{R_S + R_{SE}}{R_S} + \frac{F_2 - 1}{\left(\dfrac{R_S}{R_S + R_{SE}}\right)} =$$

$$\frac{50 + 150}{50} + \frac{2 - 1}{\left(\dfrac{50}{50 + 150}\right)} = 8 = 9.03dB$$

Optimize with input step-up transformer



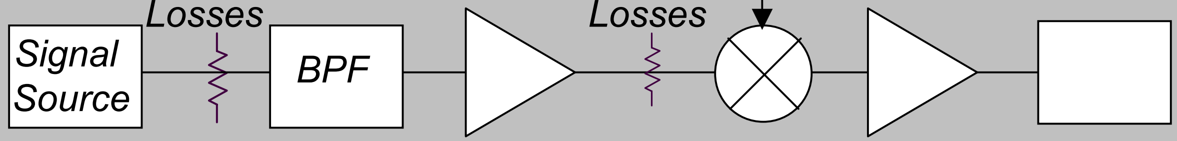
$$F_{1-2} = F_1 + \frac{F_2 - 1}{G_1} = 1 + \frac{2 - 1}{1} = 2 = 3dB$$

Resistors in input matching networks adversely impact noise figure!

Cascade Formula for Noise Figure (Cont'd)

Example, Receiver Cascade

Component No.:		(1) Line Losses	(2)	(3) Preamp	(4) Line Losses	(5) Mixer	(6) IF Ampl	(7) Rcvr



		(1)	(2)	(3)	(4)	(5)	(6)	(7)
G (dB)		-0.1	-0.3	20	-1	-6	30	
T, T_eºK	290	320	320		320	380		
F (dB)				3			1	15
G (Ratio)		0.977	0.933	100	0.794	0.25	1000	-
$G_{1\text{-}k}$ (Ratio)		0.977	0.912	91.15	72.38	18.17	18166	
F* (Ratio)		1.026	1.079	2	1.286	4.906	1.259	31.62

$$F_{1-7} = 1.026 + \frac{1.079-1}{0.977} + \frac{2-1}{0.912} + \frac{1.286-1}{91.154} + \frac{4.906-1}{72.376} + \frac{1.259-1}{18.166} + \frac{31.62-1}{18166} =$$

$$\underset{(1)}{1.026} + \underset{(2)}{0.081} + \underset{(3)}{1.096} + \underset{(4)}{0.003} + \underset{(5)}{0.054} + \underset{(6)}{0.014} + \underset{(7)}{0.002} = 2.278 = 3.576dB$$

* For a resistive element, $F = 1 + \frac{T_X}{T_O}\left(\frac{1}{G} - 1\right)$

System Noise Temperature Formulations

Derivation

In low-noise systems and with low source temperature, $T_S \ll T_O$, treatment in terms of noise temperature[7], rather than noise figure, is frequently preferred. Derivation follows.

•Output noise power level, N_O, for system operating from source at T_O is:

$$N_O = FGkT_OB = GkT_OB + (F-1)GkT_OB$$

•With a source temperature at T_S,

$$N_O = GkT_SB + (F-1)GkT_OB = GkB(T_S + T_e)$$

•Where $T_e = (F-1)T_O$ is defined as the effective input noise temperature of the receiver*.

•The total equivalent noise temperature of the system referenced to its input terminals is $T_{SYS} = T_S + T_e$.

**T_e is more concise than F in defining the noise performance of a receiver in that it is independent of source temperature.*

System Noise Temperature Formulations, (Cont'd)

System Sensitivity Analysis Using Noise Temperature

Signal source is antenna, pointed at sky with effective temperature, T_a=30ºK. Receiver system noise figure is NF=0.5dB, F=1.122:1.

- The equivalent noise temperature of the receiver is:
  - $T_e=(1.22-1)290=35.4$ºK
- Therefore effective system noise temperature is:

  $T_{SYS}=T_a+T_e=(30+35.4)=65.4$ ºK
- Equivalent input system noise is*:

  $N_S= -174(\text{dBm/Hz})+10\log(65.4/290)= -180.5$ dBm/Hz

**The term: -174(dBm/Hz) is the thermal level for 290ºK; the term: +10 log(65.4/290), corrects for a system noise temperature of 65.4 ºK*

System Noise Temperature Formulations (Cont'd)

Cascade Formulation

For cascade of n devices the noise temperature of the cascade is,

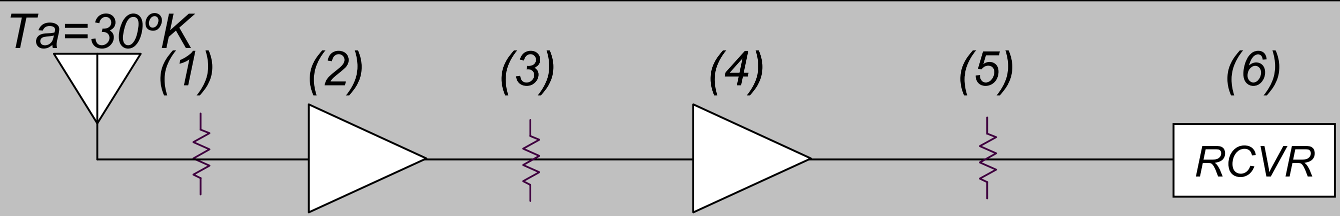
$$T_e = T_{e1-n} = (F_{1-n} - 1)T_O = \left(F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots + \frac{F_n - 1}{G_1 G_2 \ldots G_{n-1}} - 1 \right) T_O$$

$$= T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \ldots + \frac{T_n}{G_1 G_2 \ldots G_{n-1}}$$

The Noise Temperature of the system, including the source at T_S is; $T_{SYS} = T_S + T_e$

System Noise Temperature Formulations (Cont'd)

Example Receiver Analysis



	Line Loss	Preamp	Line Loss	Postamp	Line Loss	
Gain (dB)	-0.2	14	-0.05	20	-0.5	-
Gain Ratio	0.955	25.119	0.989	100	0.891	-
F(dB)	-	-	-	-	-	3
T(ºK)	320	-	320	-	320	-
Te(ºK)*	15.081	35	3.705	125	39.046	290

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \frac{T_{e4}}{G_1 G_2 G_3} + \frac{T_{e5}}{G_1 \ldots G_4} + \frac{T_{e6}}{G_1 \ldots G_5} =$$

$$15.081 + \frac{35}{0.955} + \frac{3.705}{0.955x25.119} + \frac{125}{0.955x25.119x0.989} + \frac{39.046}{0.955x25.1119x0.989x100} +$$

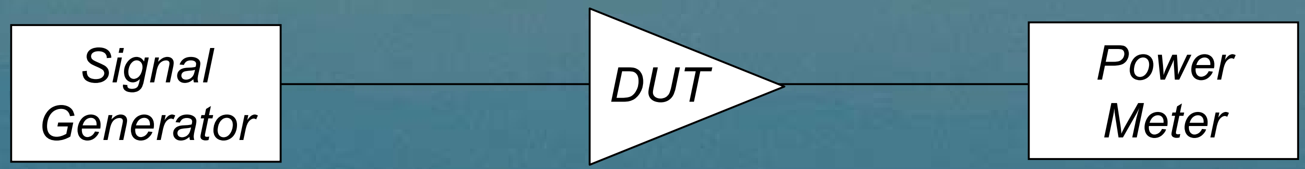
$$\frac{290}{0.955x25.119x0.989x25.1x100} = 15.081 + 36.649 + 0.154 + 5.269 + 0.017 + 0.131 = 57.309^O K$$

(1) (2) (3) (4) (5) (6)

$$* \text{For a resistive element, } T_e = T\left(\frac{1}{G} - 1\right)$$

Measurement of Noise Figure

Signal Generator Method[5,7]



Procedure

- Tune Signal generator over frequency to measure output variation of power
- From data, determine B (Slide 22)
- Turn signal generator off, and note output noise power level, $N_O = FGkT_OB$
- Turn signal generator on and tune to frequency of maximum G; adjust its level to S_i to just double output indication, to 2 N_O
- Then $GS_i = FGkT_OB$, and $F = S_i/kT_OB$
- $NF(dB) = S_i(dBm) + 114 - 10 \log B(MHz)$

Example

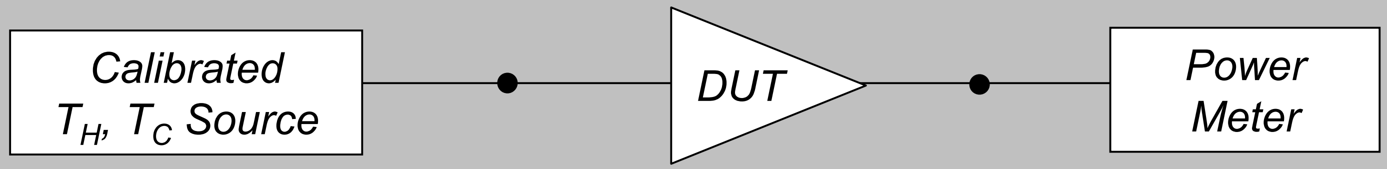
B=0.5 MHz, (-3 dB MHz)

S_i=-90 dBm

Therefore: NF(dB)=-90+114-(-3)=27 dB

Measurement of Noise Figure (Cont'd)

Calibrated Noise Source Method (Y Factor)[7]



$kT_HB \longrightarrow$

$kT_CB \longrightarrow$

$N_{OH}=FGkT_OB+(T_H-T_O)kBG$

$N_{OC}=FGkT_OB+(T_C-T_O)kBG$

$$\therefore Y=\frac{N_{OH}}{N_{OC}}=\frac{FT_O+T_H-T_O}{FT_O+T_C-T_O};\quad F=\frac{\left(\frac{T_H}{T_O}-1\right)-Y\left(\frac{T_C}{T_O}-1\right)}{Y-1}$$

Example:

T_H=10,290ºK (argon source), T_C=300ºK

Measured Y factor: Y=9 dB (7.94:1)

Then,

$$F=\frac{\frac{10290}{290}-1-7.94\left(\frac{300}{290}-1\right)}{7.94-1}=4.94;\ NF(dB)=10\log(4.94)=6.9dB$$

Measurement of Noise Figure (Cont'd)

Comparison of Measurement Methods

	Signal Generator Method	**Noise Source Method**
Advantages	Accurate results even with significant out-of-band responses Useful for measurement of devices with high noise figure	Does not require separate determination of B Measurement is simple and straight-forward Lends itself to automatic measurements
Disadvantages	Requires separate measurement of B Not easily adaptable to automatic measurements	Capabilities of available noise sources don't allow measurement of devices with high noise figure Requires correction for presence of out-of-band responses

Presentation Summary

- The background and application of the concepts of Noise Figure and Noise Temperature for characterizing the fundamental limitations on the absolute sensitivity of receivers were set forth in a step-by step approach and illustrated with examples to provide insight into the concepts.
- The origin of the major components of receiver noise, and their characteristics were summarized.
- Sample Noise Figure and Noise Temperature analyses of receiver systems were illustrated.
- The basic methods of measurement of Noise Figure and Noise Temperature were described and compared.
- References and a Bibliography follow. The Bibliography is intended to serve as a basis for further study.

References

(1) Friis, H.T., Noise Figures of Radio Receivers, Proc. Of the IRE, July, 1944, pp 419-422
(2) Johnson, J. B. *Thermal Agitation of Electricity in Conductors,* Physical Review, July, 1928, pp 97-109
(3) Nyquist, H. *Thermal Agitation of Electric Charge in Conductors*, Physical Review, July, 1928, pp. 110-113
(4) Joshua Israelsohn. *Noise 101,* EDN January 8, 2004, PP. 41-48.
(5) Stanford Goldman. *Frequency Analysis, Modulation and Noise.* McGraw-Hill Book Company, Inc., 1948
(6) Van der Ziel, Aldert. Noise:Sources, Characterization, Measurement, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1970
(7) D. C. Hogg and W. W. Mumford, The Effective Noise Temperature of the Sky, The Microwave Journal, March 1960, pp 80-84.

Bibliography

<u>*Agilent Application Notes*</u>

The following 5 Agilent Application Notes are highly recommended for study and reference. Together, they provide excellent material on noise figure and noise temperature. They progress from the background of noise and receiver sensitivity, through a summary of computer-aided design of amplifiers for optimum noise figure, description of measurement setups and techniques, including analysis of measurement uncertainty. They include comprehensive lists of references. They can be ordered from the Agilent web site.

•*Fundamentals of RF and Microwave Noise Figure Measurements,* Application Note 57-1

•*Noise Figure Measurement Accuracy- The Y-Factor Method,* Application Note 57-2

•*10 Hints for Making Successful Noise Figure Measurements,* Application Note 57-3

•*Noise Figure Measurement of Frequency Converting Devices, Using the Agilent NFA Series Noise Figure Analyzer*, Application Note 1487

•*Practical Noise-Figure Measurement and Analysis for Low-Noise Amplifier Designs*, Application Note 1354.

Bibliography (Cont'd)

<u>Text Books</u>

van der Ziel, Aldert. *Noise: Sources, Characterization, Measurement*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1970.-*Everything you ever wanted to know about noise but were afraid to ask. The classic text on the subject. Covers noise and noise models for devices, at a high technical level*

Stanford Goldman. *Frequency Analysis, Modulation and Noise.* McGraw-Hill Book Company, Inc., 1948. I

Norman Dye and Helge Granberg. *Radio Frequency Transistors, Principles and Applications,* EDN Series for Design Engineers, Motorola Series in Solid State Electronics. *Contains examples of noise figure analysis models of amplifier stages*

Bibliography (Cont'd)

Background

Johnson, J. B. *Thermal Agitation of Electricity in Conductors,* Physical Review, July, 1928, pp 97-109. *This is the classic paper on measured thermal noise in resistors. It showed that the mean square thermal noise voltage at the terminals of a resistor were proportional to the magnitude of the Ohmic value of its resistance and to its absolute temperature. The noise voltage is referred to as "Johnson noise" in his honor.*

•Nyquist, H. *Thermal Agitation of Electric Charge in Conductors*, Physical Review, July, 1928, pp. 110-113. *This is the classic paper companion paper to the Johnson Paper which related the Johnson noise to the fundamental laws of thermodynamics and arrived at the famous, KTB*

•Friis, H.T., *Noise Figures of Radio Receivers*, Proc. Of the IRE, July, 1944, pp 419-422. *This is the classic paper on Noise Figure. It should be reviewed as an excellent example of a concise presentation of an important concept.*

Bibliography (Cont'd)

Background (Cont'd)

Harold Goldberg. *Some Notes on Noise Figure,* Proc. Of the IRE, October, 1948, pp 1205-1214. *The paper is a tutorial based on the Friis paper. It carefully explores the basics concepts of noise figure and illustrates their application, progressing from elementary networks to low-noise vacuum tube amplifier configurations. The paper's approach served as a model for this presentation.*

Kurt Stern. *"Fundamentals of Electrical Noise",* Presentation to the IEEE MTT Section of Long Island NY, January, 2004. *This reviews the basics of noise figure and its measurement. It includes detail of automatic measurement setups and of available test equipment. The presentation is available on the web site for the LI NY MTT Chapter of the IEEE*

Joshua Israelsohn. *Noise 101,* EDN January 8, 2004, PP. 41-48. *Good summary on thermal, shot, and jitter noise.*